Transmission-Line Low-Profile Antennas

BY JOHN S. BELROSE,* VE2CV/VE3DRC

Limited-space antennas are of ever increasing interest to the amateur. Dr. Belrose is no newcomer to the subject, as material contained in his QST *article*[1] *on mobile antennas has been included in* The ARRL Antenna Book *for a number of years. The topic of this article is one form of a class of low-profile, transmission-line antennas popularly called the DDRR antenna. Both theoretical considerations and experimental data are included, but the practical design aspects are not omitted. The material contained here should be of use in constructing this interesting antenna both for hf and vhf purposes.*

ONE FORM of electrically short antenna is a transmission line fed against a ground plane, as shown in Fig. 1. This type of antenna has been used in a variety of configurations but in the most common one, l is approximately a quarter wavelength long. Z_T is an open circuit or a very small capacitance which can be used to tune the antenna to obtain a resonant input over a narrow band of frequencies. The transmission line, rather than being straight as shown in Fig. 1, is formed into a circle, a hexagonal, or a spiral configuration. (See Fig. 2.) Boyer[2], the inventor of this ring transmission-line type of antenna, described the radiation from it in terms of the directional discontinuity of the curved transmission line. He believed this was an important parameter in determining the radiation properties of the antenna. Hence the name directional discontinuity ring radiator (DDRR). This is perhaps not a satisfactory name, since the radiation efficiency is almost independent of the shape of the horizontal part of the antenna. Burton and King[3] analyzed the antenna as an open center-driven loop over a ground screen, which is also not a satisfactory equivalent for the antenna. While an accurate solution for a *circular* loop driven by a generator in series with it is available for a loop over a perfectly conducting ground plane, there is no accurate solution when the loop is driven in the manner of the hula hoop. Furthermore, the formulas developed in the original article by Burton and King were for two conductors in space, and the radiation resistance given in that article was too high by a factor of two.

Dome[4] and the author of this article feel they have correctly analyzed the antenna. It should be treated as a short grounded vertical radiator having a horizontal or flat top (an L-type antenna) to provide quarterwave resonance. *Most of the radiation is from the vertical stub part of the antenna. Only a small part of the radiated field is caused by the current flowing in the horizontal part.* When $h << \lambda$, the out-of-phase current in the bottom wire (or in the image of the antenna when it is mounted above a ground plane, as is generally the case) almost exactly cancels the radiation from the top wire.

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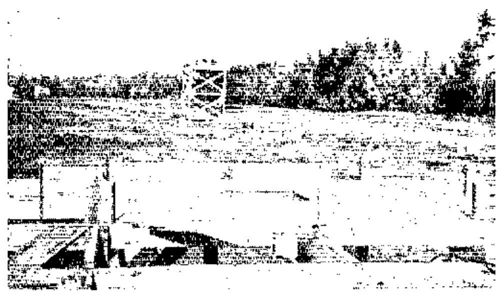
[1] For this and subsequent references, see bibliography at the end of this article.



From the above discussion, it is suggested that the name directional discontinuity ring radiator (DDRR) should be abandoned. The Northrupt Corporation later redefined DDRR to stand for directly driven resonant radiator, which is also a rather inappropriate name since a vast class of antennas are resonant and directly driven. Directly driven ring radiator is also not a suitable name, since only a small part of the radiated field is caused by the current flowing in the ring. While it might be possible to find a name that would retain the DDRR designation, the author of the present article is of the opinion that the antenna should be renamed the hula-hoop transmission-line (HHTL) radiator, since this name better identifies the form of the antenna and the class to which it belongs.

Calculation of Radiation Resistance

As mentioned in this foregoing, the HHTL

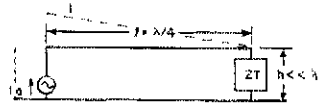


Fig. 1 — Basic quarter-wave electrically short transmission-line antenna.

antenna may best be described as being a short grounded vertical radiator having a horizontal or flat-top extension to provide quarter-wave resonance. Various authors have their preferred formulas for analyzing such antennas. The one used by this writer, given by Laport[5] is:

$$R_r = 0.01215\,A^2 \text{ ohms}$$

where A is the degree-ampere plot of the current distribution on the vertical part of the antenna. The current will maximize at the grounded base of the antenna (I_a) and will fall off as the cosine of the angular distance along the antenna to zero at the open end of the radiator. Since the current will be almost uniform over the vertical part of the radiator, $A = G_v$ degree-amperes (for $I_a = 1$), where G_v is the electrical height of the antenna in degrees of the vertical portion of the antenna of height h. For the antenna dimensions given by Blocker[6] in Fig. 3, at $f = 3.9$ MHz ($\lambda = 252.3$ feet), $h = 2.2$ feet:

$$G_v = \frac{h}{\lambda}(360^\circ) = \frac{2.2}{252.3}(360^\circ) = 3.14^\circ$$

$$\text{and } R_r = .01215\,(3.14)^2 = 0.12 \text{ ohms}$$

This formula ignores the radiation from the horizontal part of the antenna, but this is small as we shall see. The corrected formula for radiation resistance of the hula-hoop antenna which emerged after extensive discussion between Messrs. Trulio and Kriz of the Northrupt Ventura Company and Drs. Burton and King (King, private communication, 1969) is:

$$R_r = 10\,(\beta c)^2 + 0.45\,(\beta c)^2 \text{ ohms}$$

where $\beta = 2\pi/\lambda$, λ = wavelength in meters and $c/2$ is the height of the radiator (meters) above the ground plane. The term $10\,(\beta c)^2$ is the contribution from the vertical stub over the ground plane and $0.45\,(\beta c)^4$ is the radiation from the open-ended horizontal part of the antenna. In our terminology, the height of the antenna is h and $c = 2h$. Substituting into the foregoing equation gives:

$$R_r = 1650\,(h/\lambda)^2$$

This equation is almost exactly that given by Dome[4].

Radiation Efficiency

It is clear that the radiation resistance of the HHTL antenna is very low, and unless the loss resistances (ground-loss and conductor-loss resistances) are also very low, the radiation efficiency will be rather poor. The radiation efficiency of the antenna is the radiation resistance R_r divided by the total antenna resistance R_a or:

$$r = \frac{R_r}{R_a}(100)\%$$

where R_a equals the sum of the radiation resistance (R_r), the conductor-loss resistance (R_c) and the ground-loss resistance (R_g).

Matching the HHTL Transmission-Line Antenna

The tapped resonant quarter-wavelength line and its equivalent circuit are shown in Fig. 4. Transmission lines have been analyzed by many authors; the reference this writer prefers is that of Jordan[7]. As the tap point is moved from the shorted end toward the open end of the line, the impedance seen at the tap point is a pure resistance that varies from a very low value to quite a high value:

$$R_s = \frac{2Z_0^2}{Rl}$$

where Z_0 = surge impedance of the transmission line in ohms, R is the total resistance per loop meter in ohms, and l is the length of the transmission line in meters.

The input impedance at a distance x measured along the line from the shorted end is:

$$R_x = \frac{2Z_0^2}{Rl}\sin^2\beta x$$

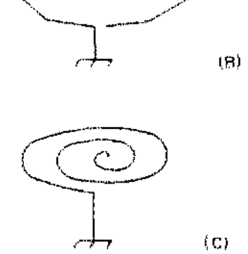


Fig. 2 — Electrically small antenna with top loading curved into a circle, a hexagon, or a flat spiral (length $l \sim d/4$).

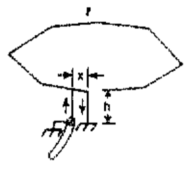


Fig. 3 — A variation of the HHTL antenna, hexagonal in shape which is easier to construct than the circular loop figuration, showing how the feed is tapped onto the transmission line at a distance x for the required impedance match (nominally $R_x = 51$ ohms).

where βx is the electrical distance in degrees of the tap point at distance x. The Q factor is given by:

$$Q = \frac{2\pi f_o Z_o}{Rv}$$

where f_o is the resonant frequency of the $\lambda/4$ line in Hz and v is the velocity of propagation down the line ($\sim 3 \times 10^8$ m/s). The Q factor of a transmission line is equal to the inductive reactance per unit length to the resistance per unit length:

$$Q = \frac{2\pi f_o L}{R}$$

The surge impedance of the transmission line can be calculated from standard formulas. For a single conductor over a ground plane:

$$Z_o = 138 \log_{10} \frac{2h}{\rho}$$

where h is the height of the conductor above the ground plane and ρ is the radius of the conductor. Alternatively, Z_o can be determined by measuring the total capacitance of the line (with short disconnected) and the inductance with the short in place. Measurements would have to be made from the open end of the line. The impedance would then be given by the formula:

$$Z_o = \sqrt{L/C}$$

These formulas are developed from a theoretical analysis of low-loss (but not lossless) transmission lines and can be used in approximate calculations for the parameters of the HHTL antenna. The formulas differ from those given by Dome who assumed a lossless transmission line. In this case, the resistance that would be measured at the base of the antenna (including radiation and ohmic-loss resistances) is merely transformed up to the tap point.

Neither the formulas mentioned previously nor those of Dome's article can be used to depict conditions at the tap point very accurately, as we shall see. Both are based on a transmission-line theory which does not consider progressive radiation from the line, and coupling between the shorted stub and feed-point stub is neglected. In many systems, the distance x will be small (Fig. 3) and the latter coupling will have significant effect which will be described later. Experimentally, it has been found that as the feed point is moved the impedance is not a pure resistance and the resonant frequency of the antenna is changed slightly. In order to obtain a pure resistance at the feed point, the length of the antenna must be adjusted.

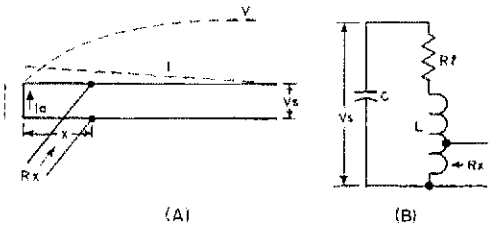


Fig. 4 — (A) Tapped quarter-wave transmission line (voltage and current distributions on the line are sketched) and (B) its equivalent resonant circuit.

Experimental Measurements

A ground plane was constructed (see title photograph) and the HHTL antenna was erected over it. The entire system was elevated about 5 feet so that impedance-measuring and transmitting equipment could be installed beneath the ground screen. This was done so that the measuring instruments would not disturb the impedance or the pattern of the antenna. The ground screen, which consisted of 2-inch welded wire mesh, was hexagonal in shape and approximately 20 feet in diameter. While the ground plane was constructed for model-antenna studies, a full-size 40-meter HHTL antenna was readily mounted on it. The HHTL antenna was constructed of four horizontal No. 18 wires spaced 1/4 inch apart and was mounted 10.8 inches above the ground plane. Frequency of operation was 7785 kHz, which was

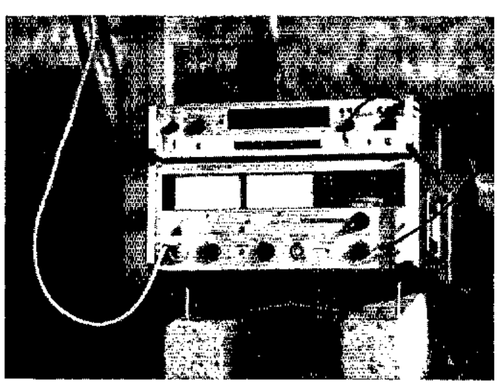


Fig. 5 — Hewlett Packard Vector Impedance Meter and Frequency Counter (for accurate bandwidth measurement) located beneath the HHTL antenna.

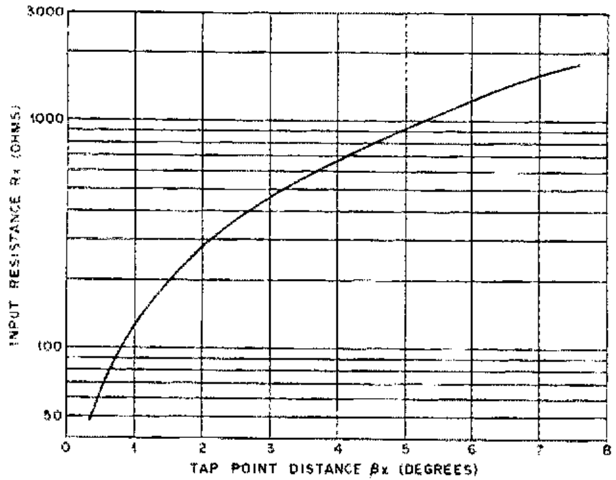


Fig. 6 — Graph based on experimentally measured data showing how the input resistance R_x varies with the tap-point distance βx in electrical degrees for a 40-m HHTL antenna described in the text.

a scale frequency for a model study of an intended low-frequency antenna system.

A Hewlett Packard Vector Impedance Meter Model 4815A and a Hewlett Packard Frequency Counter Model 5325B (for accurate bandwidth measurement) were used for antenna impedance measurements. See Fig. 5. A Drake T4XB transmitter, a calibrated directional wattmeter, a Drake Antenna Match Network MN-2000 and a Stoddart Field Intensity Meter model NM25T were used for field-strength measurements. The radiated field strength for a given transmitter power was measured at several sites (about 1000 meters from the center of the antenna), and this field was referenced to that for a true λ/4 vertical antenna mounted at the center of the ground screen, to determine the radiation effeciency of the antenna.

The HHTL antenna was found to be rather critical to tune. To obtain an input impedance of 51 ohms resistive, it was necessary to adjust both the feed point distance, *x* and the length, *l*. There was definitely some interaction. In practice, an exact match is not necessary if a matching unit is used (such as the Drake MN-2000). The distance *x* for exact resonance was found to be 1-3/4 inches and the length *l* was 372-1/4 inches. The *Q* factor was ~180 (bandwidth at 7785 kHz for a phase shift of ±45° was 43 kHz). The surge impedance was measured with a Tetronic model 1302C *L-C* meter in the following way. The grounded stub section was ungrounded and the total capacitance of the antenna was measured between the open end of the antenna and the ground plane. It was found to be 113 pF. The stub section of the antenna was grounded and the inductance of the shorted quarter wave section was measured to be 9.7 μH. The surge impedance is given by:

$$Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{9.7 \times 10^{-6}}{113 \times 10^{-12}}} = 293 \text{ ohms}$$

The measured radiation efficiency was determined by dividing the square of the field strength for the HHTL radiator by the square of the field strength for the quarter-wave vertical radiator. Assuming an efficiency of 90 percent for the quarter-wave ground plane, the measurements indicated the HHTL efficiency was approximately 11 percent.

Theoretical Analysis of Experimental Data

A theoretical analysis of this antenna follows:

f = 7785 kHz
λ = 126.3 feet (38.53m)
h = 10.8 inches (2.56°)
l = 372.25 inches (0.245λ)
x = 1.75 inches (0.415°)
Q ~ 180 (BW = 43 kHz)

$$G_v = \frac{(10.8)(360°)}{(126.4)(12)} = 2.56°$$

$$R_r = .01215\, G_v^2 = (.01215)(2.56)^2 = .08 \text{ ohms}$$

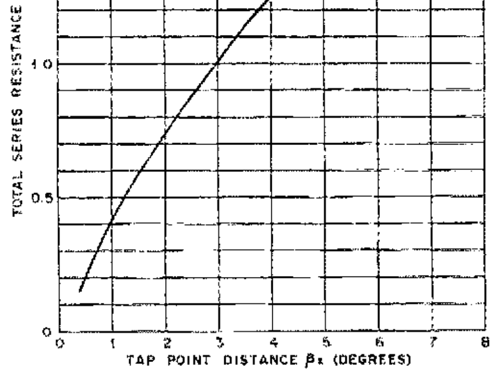


Fig. 7 — Graph showing how the apparent value of the total series resistance of the quarter-wave transmission line *Rl* (ohms) varies with tap-point distance (βx) in electrical degrees for a 40-m HHTL antenna.

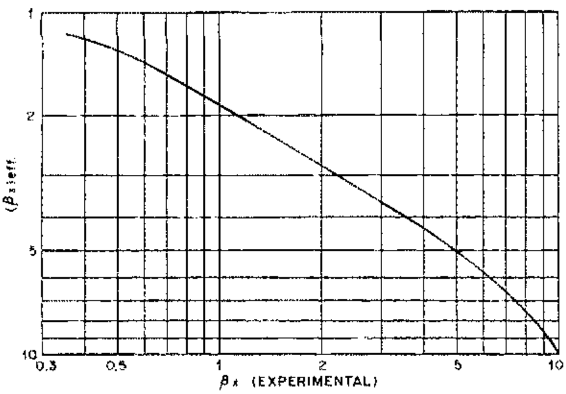


Fig. 8 — β_{eff} as a function of βx.

The calculated radiation resistance was:

$$R_r = 1650 \left[\frac{10.8}{(126.4)\,(12)}\right]^2 = .083 \text{ ohms}$$

Using the measured radiation efficiency and:

$$\eta = \frac{R_r}{R_a}\,(100)\%$$

gives the total resistance referred to the base of the ground stub of the antenna (R_a – radiation and loss resistances) as 0.73 ohm.

The graph in Fig. 6 shows how the input resistance (R_x) varied with the tap distance βx (in degrees). With some rearrangement, the equation for R_x can be employed to calculate the tap-point distance x for a given input resistance if the line-loss resistance (Rl) is known. Also, if the distance is measured experimentally the loss resistance can be deduced. In Fig. 7 is shown the manner in which this calculated value for loss resistance (Rl) varies with the tap-point distance (βx in degrees). When βx is small, Rl is a function of βx, whereas it should be independent. Consequently, the theoretical formula for R_x does not very accurately predict the impedance at the tap point.

The equation for the Q factor for a $\lambda/4$ transmission line can be rearranged and employed to estimate the loss resistance R if Q is measured.

$$R = 2\pi f_o Z_o = \frac{2\pi(7.785 \times 10^6)(293)}{(180)\,(3 \times 10^8)} = 0.26 \text{ ohms/m}$$

Since the length of the transmission line l is 9.44 m, Rl is 2.45 ohms. This same value of Rl is deduced from:

$$Q = \frac{2\pi f_o L l}{Rl}$$

$$\text{or } Rl = \frac{2\pi f_o L l}{Q} = \frac{(2\pi)\,(7.785 \times 10^6)\,(9.7 \times 10^{-6})}{180}$$

$$= 2.63 \text{ ohms}$$

The Q factor was found to be independent of the tap position, and using this value of Rl and the equation for R_s, the impedance at the open end of the transmission line can be found from:

$$R_s = \frac{2(293)^2}{2.5} = 68.68 \times 10^3 \text{ ohms}$$

The voltage at the open end of the transmission line, for a transmitter power of 1000 watts, is given by

$$V_s = \sqrt{PR} = \sqrt{(1000)(68.68 \times 10^3)} =$$

$$8.29 \text{ kV (rms)}$$

The peak voltage is $\sqrt{2}$ times the rms voltage or:

$$V_s \text{ (peak)} = 11.72 \text{ kV (peak)}$$

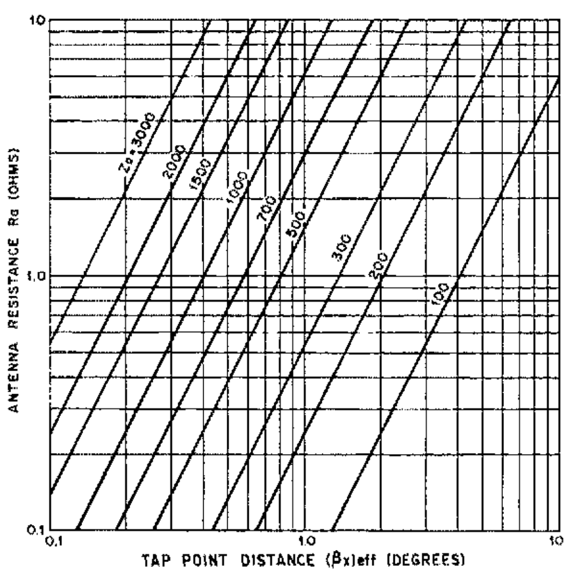


Fig. 9 — Calculation relation for a feed-line input impedance Rx equal to 50 ohms between tap-point distance $(\beta x)_{eff}$ in degrees and antenna resistance Ra in ohms for various values of line surge impedance (Z_o) ohms.

In the case of the normal quarter-wave impedance-matching application employing transmission lines (Fig. 4), currents flowing in the input transmission line do not couple directly with currents flowing in the stub. However, in the HHTL application (Fig. 3), currents flowing in these elements couple when βx is small because the feed-point stub is parallel to the shorted stub and separated by only a small distance. A more rigorous analysis for a lossy line, including radiation from it and coupling between the driven-stub element and the shorted-stub element, should be made to obtain a more accurate expression for impedance matching. However, in practical applications the distance x can be easily obtained experimentally (adjustments are made for minimum SWR). Therefore, a rigorous analysis is not attempted here but instead an empirical approach is taken to provide data suitable for design applications.

Empirical Approach to Interpretation of Experimental Measurements

The effect of coupling between the driven and shorted stub elements will depend on spacing βx and on currents (hence the impedances) in the shorted and driven elements. The experimental measurements show that the effect of coupling is to reduce the actual value of βx (when βx is small), required for impedance step up to a value less than that calculated theoretically. If we consider, as did Mr. Dome,[4] that the antenna resistance R_a is impedance transferred to R_x by tapping the line at distance βx degrees, the equation

$$R_x = Z_o^2/R_a \sin^2 \beta_x$$

can be written as

$$R_x = Z_o^2/R_a \sin^2 (\beta x)_{eff}$$

where $(\beta x)_{eff}$ is the effective value of βx such that equality holds for a given impedance step up. The graph in Fig. 8 shows the experimentally deduced dependence of $(\beta x)_{eff}$ on βx. This graph shows a result anticipated. As βx increases (for $\beta x > 5°$) $(\beta x)_{eff} \sim \beta x$ indicating coupling effects are becoming small.

To use this empirically derived graph in actual antenna design and development we can proceed as follows: (1) estimate the value of antenna resistance R_a based on experience, previous measurements or guess; (2) use the foregoing equation and the desired value of R_x (usually 50 ohms) and solve this equation for $(\beta x)_{eff}$. This is facilitated by use of the graph in Fig. 9, which gives the antenna resistance as a function of $(\beta x)_{eff}$ for various values of Z_o for an input impedance of 50 ohms. The transmission-line impedance Z_o is calculated from:

$$Z_o = 138 \log_{10} \frac{2h}{\rho}$$

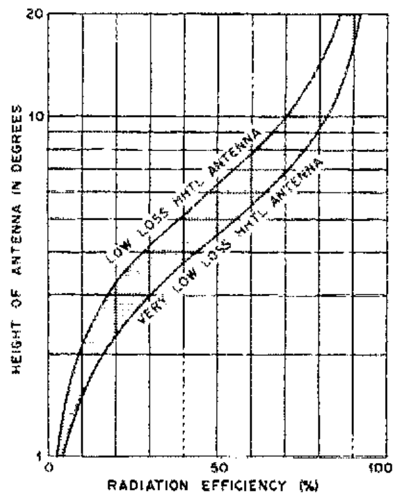


Fig. 10 — Graph showing how the radiation resistance of a HHTL antenna would be expected to vary with the electrical height of the antenna above the ground plane. The upper curve is for a low-loss antenna which would be expected to apply at hf; the lower curve is for a very low-loss antenna as might be the situation at vhf where the hula hoop is a silver plated tube and the ground screen a copper sheet.

(3) for this value of $(\beta x)_{eff}$ determine from the graph in Fig. 8 the value of βx; (4) with the feed line connected to the antenna at this distance, apply low power and adjust the antenna (x and l) for minimum SWR; (5) the experimentally determined value for βx can now be used to determine (using Fig. 8) the value of $(\beta x)_{eff}$ that applies to the particular antenna under development; (6) with this value for $(\beta x)_{eff}$, the graph in Fig. 9 can be used to calculate the actual value of antenna resistance R_a; (7) using this value of R_a and the calculated value for the radiation resistance R_r from:

$$R_r = 0.01215 A^2 \text{ ohms}$$

or

$$R_r = 1650 (h/\lambda)^2$$

the antenna efficiency can be determined from:

$$r = \frac{R_r}{R_a} (100)\%$$

and compared in Fig. 10 with the efficiency expected for the particular antenna configuration used.

Conclusions

Transmission-line radiators are low-profile antennas, which have a radiation efficiency, although rather low, that is comparable with a mobile whip antenna. A 110-inch center-loeaded whip whould have a radiation efficiency of ~ 15% at 7785 kHz.

The radiation efficiency is therefore rather high for the physical height of the antenna. Since the radiation resistance for short antennas varies approximately with the height of the antenna above the ground plane squared, only a small increase in the height of the antenna will make a very great difference in the radiation efficiency. The shaded lines in Fig. 11 show how the estimated radiation efficiences, based on antenna parameters measured and modeled by the author and his colleagues, would be expected to change with the height of the antenna above the ground plane. A radiation efficiency of about 75% can be achieved for an antenna which is 10 electrical degrees above the ground plane (about 7 feet at 3800 kHz or 3-3/4 feet at 7250 kHz).

In summary, the principal advantages of the HHTL antenna are: (1) its small physical height makes it adaptable for mobile operation at least at the higher frequencies or it could be mounted o the roof of an apartment building or a ranch-style house and not be visible from the street; (2) the matching to transmitter, although critical, is adjusted easily; (3) the antenna is at dc ground (low noise pickup due to static build up); (4) if the top wire is wound in a spiral as shown in Fig. 2C, the physical dimensions of the antenna can be made very small (a 20-meter antenna might have a flat top 3 × 3 feet and the height of the radiator about 12 to 24 inches). The disadvantages are: (1) very high Q for low loss when $h < 10°$ and hence narrow bandwidth and large voltages between the open end of the antenna and ground; (2) the loss must be kept very small for acceptable efficiency; (3) the effects of snow and ice covering the antenna have not been investigated but are expected to cause detuning. Because of the high Q factor, it is essential that the HHTL antenna be operated at resonance. If the open-circuit end of the antenna is terminated at a capacitor to obtain resonance over a narrow band of frequencies, the capacitance must be very small since the current flowing in it will in part cancel the field from the shorted stub. A full-wave transmission line radiator (see Fig. 11) has certain advantages in the upper hf range and at vhf where the size is practical, since both ends of the antenna are grounded. If the antenna is made of tubing it can be self supporting, with no support (and associated insulator problems) at the midpoint of the horizontal part of the radiator where the voltage is highest. The half-wave transmission-line radiator, like the quarter-wave one, can be bent into a circle or a spiral and tapped to give an impedance match to 51-ohm coaxial line.

QST

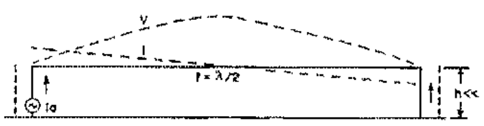


Fig. 11 — Basic half-wave, electrically short, transmission-line antenna.

Acknowledgement

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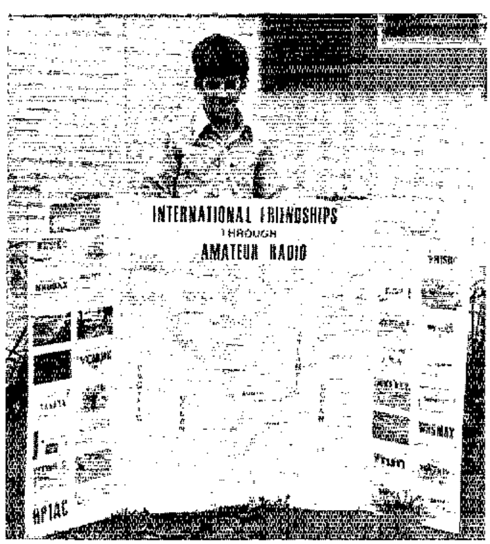
Strays



WB5KNF, Michael Garon, presents the display which won him first prize in his school's social science contest. Michael is active around 7,260 kHz almost every night.